
Section II: Free-Response Questions

This is the free-response section of the 2014 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP[®] Calculus AB Exam

SECTION II: Free Response

2014

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time

1 hour, 30 minutes

Number of Questions

6

Percent of Total Score

50%

Writing Instrument

Either pencil or pen with black or dark blue ink

Weight

The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions

2

Time

30 minutes

Electronic Device

Graphing calculator required

Percent of Section II Score

33.3%

Part B

Number of Questions

4

Time

60 minutes

Electronic Device

None allowed

Percent of Section II Score

66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name

First letter of your first name

2. Date of birth

Month Day Year

3. Six-digit school code

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.

No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as `fnInt(X2, X, 1, 5)`.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I

Form Code 4JBP6-S

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CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

GO ON TO THE NEXT PAGE.

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t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

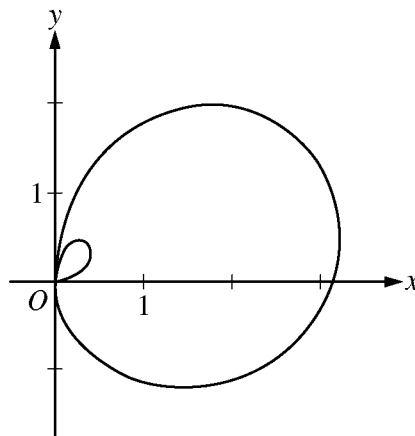
1. Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \leq t \leq 2.4$ hours, as shown in the table above. For $0 < t \leq 2.4$, $v(t) > 0$.
- (a) Use the data in the table to approximate Ruth's acceleration at time $t = 1.4$ hours. Show the computations that lead to your answer. Indicate units of measure.

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- (b) Using correct units, interpret the meaning of $\int_0^{2.4} v(t) dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t) dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

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- (c) For $0 \leq t \leq 2.4$ hours, Ruth's velocity can be modeled by the function g given by $g(t) = \frac{24t + 5\sin(6t)}{t + 0.7}$. According to the model, what was Ruth's average velocity during the time interval $0 \leq t \leq 2.4$?

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- (d) According to the model given in part (c), is Ruth's speed increasing or decreasing at time $t = 1.3$? Give a reason for your answer.



2. Consider the polar curve defined by the function $r(\theta) = \theta \cos \theta$, where $0 \leq \theta \leq \frac{3\pi}{2}$. The derivative of r is given by $\frac{dr}{d\theta} = \cos \theta - \theta \sin \theta$. The figure above shows the graph of r for $0 \leq \theta \leq \frac{3\pi}{2}$.
- (a) Find the area of the region enclosed by the inner loop of the curve.

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- (b) For $0 \leq \theta \leq \frac{3\pi}{2}$, find the greatest distance from any point on the graph of r to the origin. Justify your answer.

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- (c) There is a point on the curve at which the slope of the line tangent to the curve is $\frac{2}{2-\pi}$. At this point, $\frac{dy}{d\theta} = \frac{1}{2}$. Find $\frac{dx}{d\theta}$ at this point.

END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON
PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

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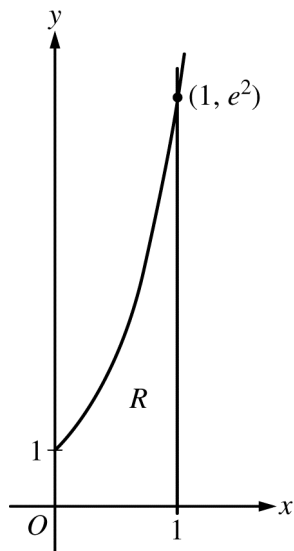
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NO CALCULATOR ALLOWED



3. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of $y = f(x)$ and the vertical line $x = 1$, as shown in the figure above.
- (a) Write an equation for the line tangent to the graph of f at $x = 1$.

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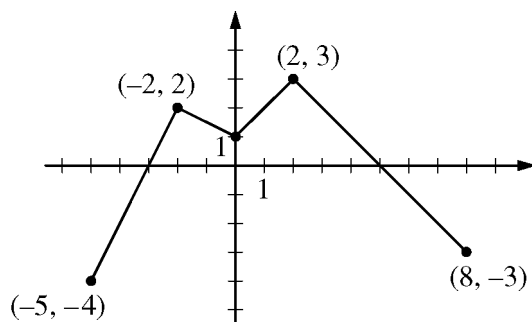
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NO CALCULATOR ALLOWED

(b) Find the area of R .

(c) Region R forms the base of a solid whose cross sections perpendicular to the y -axis are squares. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

NO CALCULATOR ALLOWED

Graph of f

4. The continuous function f is defined on the interval $-5 \leq x \leq 8$. The graph of f , which consists of four line segments, is shown in the figure above. Let g be the function given by $g(x) = 2x + \int_{-2}^x f(t) dt$.

(a) Find $g(0)$ and $g(-5)$.

(b) Find $g'(x)$ in terms of $f(x)$. For each of $g''(4)$ and $g''(-2)$, find the value or state that it does not exist.

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NO CALCULATOR ALLOWED

(c) On what intervals, if any, is the graph of g concave down? Give a reason for your answer.

(d) The function h is given by $h(x) = g(x^3 + 1)$. Find $h'(1)$. Show the work that leads to your answer.

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5. A toy train moves along a straight track set up on a table. The position $x(t)$ of the train at time t seconds is measured in centimeters from the center of the track. At time $t = 1$, the train is 6 centimeters to the left of the center, so $x(1) = -6$. For $0 \leq t \leq 4$, the velocity of the train at time t is given by $v(t) = 3t^2 - 12$, where $v(t)$ is measured in centimeters per second.

(a) For $0 \leq t \leq 4$, find $x(t)$.

(b) Find the total distance traveled by the train during the time interval $0 \leq t \leq 4$.

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- (c) A toy bus moving on the same table has position given by $(x(t), y(t))$. Here, $x(t)$ is the function found in part (a), and $y(t) = 2 + 12\sin\left(\frac{t}{4}\right)$ is the distance from the bus to the train track, in centimeters. Write, but do not evaluate, an integral expression that gives the total distance traveled by the bus during the time interval $0 \leq t \leq 4$.

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NO CALCULATOR ALLOWED

6. Let $a_n = \frac{1}{n \ln n}$ for $n \geq 3$.

(a) Let f be the function given by $f(x) = \frac{1}{x \ln x}$. For $x \geq 3$, f is continuous, decreasing, and positive. Use the integral test to show that $\sum_{n=3}^{\infty} a_n$ diverges.

(b) Consider the infinite series $\sum_{n=3}^{\infty} (-1)^{n+1} a_n = \frac{1}{3 \ln 3} - \frac{1}{4 \ln 4} + \frac{1}{5 \ln 5} - \cdots$. Identify properties of this series that guarantee the series converges. Explain why the sum of this series is less than $\frac{1}{3}$.

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NO CALCULATOR ALLOWED

- (c) Find the interval of convergence of the power series $\sum_{n=3}^{\infty} \frac{(x-2)^{n+1}}{n \ln n}$. Show the analysis that leads to your answer.

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STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**