

AP[®] Calculus BC Exam

SECTION II: Free Response

2015

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time

1 hour, 30 minutes

Number of Questions

6

Percent of Total Score

50%

Writing Instrument

Either pencil or pen with black or dark blue ink

Weight

The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions

2

Time

30 minutes

Electronic Device

Graphing calculator required

Percent of Section II Score

33.3%

Part B

Number of Questions

4

Time

60 minutes

Electronic Device

None allowed

Percent of Section II Score

66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name

First letter of your first name

2. Date of birth

Month Day Year

3. Six-digit school code

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.

No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as `fnInt(X2, X, 1, 5)`.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I

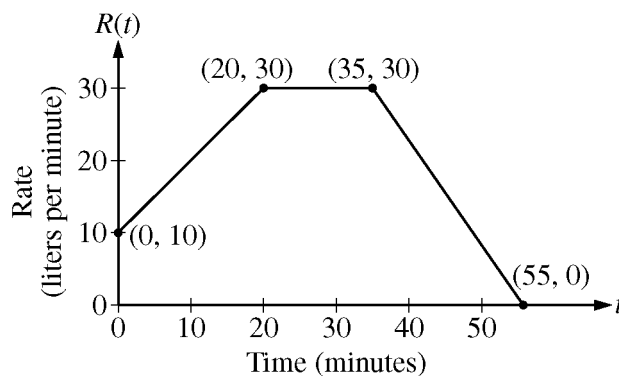
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CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

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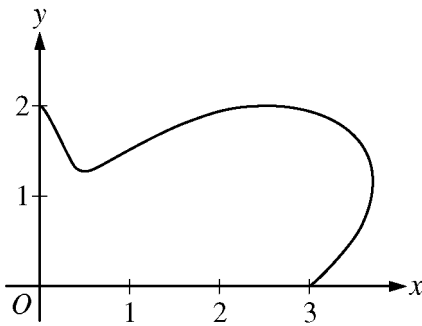
1. At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.
- (a) Find $R'(45)$. Using appropriate units, explain the meaning of your answer in the context of this problem.

- (b) How many liters of water have been pumped into the tank from time $t = 0$ to time $t = 55$ minutes? Show the work that leads to your answer.

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- (c) At time $t = 10$ minutes, water begins draining from the tank at a rate modeled by the function D , where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time $t = 55$ minutes. How many liters of water are in the tank at time $t = 55$ minutes?

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- (d) Using the functions R and D , determine whether the amount of water in the tank is increasing or decreasing at time $t = 45$ minutes. Justify your answer.



2. The figure above shows the graph of the polar equation $r = 2 + \sin(4\theta) + \cos(\theta)$ for $0 \leq \theta \leq \frac{\pi}{2}$. The derivative of r with respect to θ is given by $r'(\theta) = 4\cos(4\theta) - \sin(\theta)$.

(a) Find the area of the region bounded by the graph of r and the lines $\theta = 0$ and $\theta = \frac{\pi}{2}$.

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- (b) Find the area of the region in the first quadrant that is outside the graph of $r = 2 + \sin(4\theta) + \cos(\theta)$ but inside the graph of the circle of radius 2 centered at the origin.

- (c) Find the value of θ in the interval $0 \leq \theta \leq \frac{\pi}{2}$ that corresponds to the point on the curve $r = 2 + \sin(4\theta) + \cos(\theta)$ with greatest distance from the origin. Justify your answer.

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END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON
PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24

3. Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \leq 12$ seconds, Kathleen's velocity k , measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.
- (a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

-
- (b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t) dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t) dt$? Explain your reasoning.

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NO CALCULATOR ALLOWED

- (c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \frac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

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- (d) Write an expression for Nathan's acceleration in terms of t .

4**4****4****4****4****4****4****4****4****4****NO CALCULATOR ALLOWED**

4. In a national park, the population of mountain lions grows over time. At time $t = 0$, where t is measured in years, the population is found to be 20 mountain lions.

(a) One zoologist suggests a population model P that satisfies the differential equation $\frac{dP}{dt} = \frac{1}{4}(220 - P)$.

Use separation of variables to solve this differential equation for P with the initial condition $P(0) = 20$.

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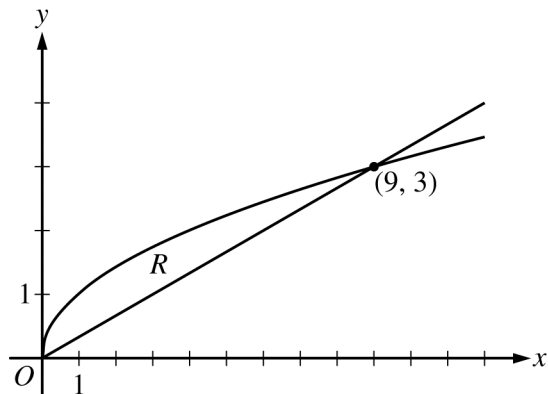
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NO CALCULATOR ALLOWED

- (b) A second zoologist suggests a population model Q that satisfies $\frac{dQ}{dt} = \frac{1}{500}Q(220 - Q)$. Find the value of $\frac{dQ}{dt}$ at the time when Q grows most rapidly.

-
- (c) For the population model Q introduced in part (b), use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $Q(10)$. Show the computations that lead to your answer.

NO CALCULATOR ALLOWED



5. Let R be the region in the first quadrant enclosed by the graphs of $g(x) = \sqrt{x}$ and $h(x) = \frac{x}{3}$, as shown in the figure above.
- (a) Find the area of region R .

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NO CALCULATOR ALLOWED

- (b) Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when R is revolved about the horizontal line $y = 4$.

-
- (c) Find the maximum vertical distance between the graph of g and the graph of h between $x = 0$ and $x = 16$. Justify your answer.

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NO CALCULATOR ALLOWED

6. The Maclaurin series for a function f is given by $\frac{x}{3} + \frac{x^2}{4} + \frac{x^3}{5} + \cdots + \frac{x^n}{n+2} + \cdots$.

(a) Use the ratio test to find the interval of convergence of the Maclaurin series for f .

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NO CALCULATOR ALLOWED

- (b) Let g be the function given by $g(x) = f(-2x)$. Find the first three terms and the general term of the Maclaurin series for g .

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- (c) The first two terms of the Maclaurin series for f are used to approximate $f(0.1)$. Given that $|f'''(x)| \leq 2$ for $0 \leq x \leq 0.1$, use the Lagrange error bound to show that this approximation differs from $f(0.1)$ by at most $\frac{1}{3000}$.

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STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**