Section II: Free-Response Questions

This is the free-response section of the 2016 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP® Calculus BC Exam

SECTION II: Free Response

2016

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B

Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4LBP6-S

CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. The curve C in the xy-plane is given parametrically by $(x(t), y(t))$, where $\frac{dx}{dt} = t \sin\left(\frac{2t}{3}\right)$ and $\frac{dy}{dt} = \cos\left(\frac{t^2}{4}\right)$ for $1 \le t \le 6$.

   (a) Find the slope of the line tangent to the curve C at the point where $t = 3$.

   (b) For $1 \le t \le 6$, what is the value of t at which the line tangent to the curve C is vertical?

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Continue problem 1 on page 5.

(c) Find the length of the curve C for $1 \leq t \leq 6$.

(d) Given $y(1) = 2$, find $y(3)$.

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t (weeks)	0	3	6	10	12
$G(t)$ (games per week)	160	450	900	2100	2400

2. A store tracks the sales of one of its popular board games over a 12-week period. The rate at which games are being sold is modeled by the differentiable function G, where $G(t)$ is measured in games per week and t is measured in weeks for $0 \le t \le 12$. Values of $G(t)$ are given in the table above for selected values of t.

(a) Approximate the value of $G'(8)$ using the data in the table. Show the computations that lead to your answer.

(b) Approximate the value of $\int_0^{12} G(t)\,dt$ using a right Riemann sum with the four subintervals indicated by the table. Explain the meaning of $\int_0^{12} G(t)\,dt$ in the context of this problem.

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Continue problem 2 on page 7.

(c) One salesperson believes that, starting with 2400 games per week at time $t = 12$, the rate at which games will be sold will increase at a constant rate of 100 games per week per week. Based on this model, how many total games will be sold in the 8 weeks between time $t = 12$ and $t = 20$?

(d) Another salesperson believes the best model for the rate at which games will be sold in the 8 weeks between time $t = 12$ and $t = 20$ is $M(t) = 2400e^{-0.01(t-12)^2}$ games per week. Based on this model, how many total games, to the nearest whole number, will be sold during this period?

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END OF PART A OF SECTION II

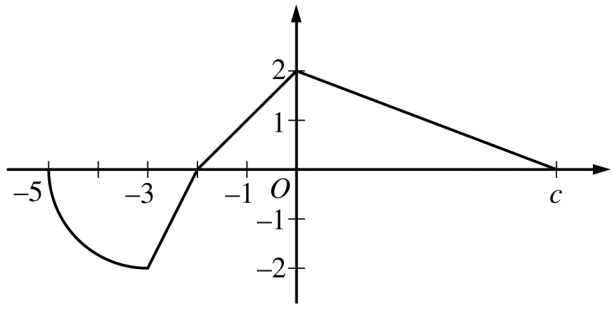
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED



Graph of f

3. The function f is defined on the interval $-5 \le x \le c$, where $c > 0$ and $f(c) = 0$. The graph of f, which consists of three line segments and a quarter of a circle with center $(-3, 0)$ and radius 2, is shown in the figure above.

   (a) Find the average rate of change of f over the interval $[-5, 0]$. Show the computations that lead to your answer.

   (b) For $-5 \le x \le c$, let g be the function defined by $g(x) = \int_{-1}^{x} f(t)\,dt$. Find the x-coordinate of each point of inflection of the graph of g. Justify your answer.

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Continue problem 3 on page 15.

(c) Find the value of c for which the average value of f over the interval $-5 \le x \le c$ is $\frac{1}{2}$.

(d) Assume $c > 3$. The function h is defined by $h(x) = f\left(\frac{x}{2}\right)$. Find $h'(6)$ in terms of c.

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NO CALCULATOR ALLOWED

4. Let R be the region bounded above by the graph of $y = e^{-x}$, below by the x-axis, on the left by the y-axis, and on the right by the vertical line $x = m$, where $m > 0$ is a constant.

   (a) Find the area of R in terms of m.

   (b) Region R is revolved around the horizontal line $y = -3$ to form a solid. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

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Continue problem 4 on page 17.

(c) Find the value of the real number $k > 1$ such that $\int_0^{\infty} k^{-x}\, dx = 2$, or show that no such k exists.

NO CALCULATOR ALLOWED

5. For $0 \le t \le 24$ hours, the temperature inside a refrigerator in a kitchen is given by the function W that satisfies the differential equation $\frac{dW}{dt} = \frac{3\cos t}{2W}$. $W(t)$ is measured in degrees Celsius (°C), and t is measured in hours. At time $t = 0$ hours, the temperature inside the refrigerator is 3°C.

   (a) Write an equation for the line tangent to the graph of $y = W(t)$ at the point where $t = 0$. Use the equation to approximate the temperature inside the refrigerator at $t = 0.4$ hour.

   (b) Find $y = W(t)$, the particular solution to the differential equation with initial condition $W(0) = 3$.

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Continue problem 5 on page 19.

(c) The temperature in the kitchen remains constant at 20°C for $0 \le t \le 24$. The cost of operating the refrigerator accumulates at the rate of \$0.001 per hour for each degree that the temperature in the kitchen exceeds the temperature inside the refrigerator. Write, but do not evaluate, an expression involving an integral that can be used to find the cost of operating the refrigerator for the 24-hour interval.

NO CALCULATOR ALLOWED

6. Let f be a function that has derivatives of all orders for all real numbers, and let $3 - 6(x-1)^2 + 5(x-1)^3 - 2(x-1)^4$ be the fourth-degree Taylor polynomial for the function f about $x = 1$.

(a) Find $f'''(1)$.

(b) Does the graph of f have a relative maximum, a relative minimum, or neither at $x = 1$? Give a reason for your answer.

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Continue problem 6 on page 21.

NO CALCULATOR ALLOWED

(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$. Find the third-degree Taylor polynomial for g about $x = 1$, and use it to approximate $g(1.1)$.

(d) Let h be the function defined by $h(x) = f'(x)$. Find the third-degree Taylor polynomial for h about $x = 1$. Verify that the graph of h does not have a point of inflection at $x = 1$.

Do not write beyond this border.

STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**